Does *Averaging-In* Work?

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Averaging-in is a type of money management scheme for trading strategies. We show in this article that averaging-in is never better than an all-in strategy where all of the money is invested at once.

Introduction

For an illustration let us consider a mean reversion trading system with an opening signal, a closing signal and sometimes an intermediate signal. Further suppose our fund can buy two shares. We have three choices for deploying that capital:

1. We can buy two shares all when our indicator reaches two. We make four dollars (two dollars per share) when it reverts to zero.
2. We can buy two shares when our indicator reaches three. We then make six dollars (three dollars per share) when it reverts to zero, but we also miss all of the opportunities where the indicator never got to three.
3. We can buy one share at the opening indicator and one at the second level. We then make five dollars every trade when the second level is hit (two dollars on the first share and three dollars on the second) but only two dollars if only the first level is hit.

Let's also assume that we always win these bets when we could get them, but that makes them fairly hard to find. (Averaging-in does have some advantages when losers are in the deck, but even there as we demonstrate later in this article, averaging-in comes up just even at best in the total profit category.).

It all comes down to how often the second signal is hit. If the second signal is hit often enough to make averaging-in pay, it is hit often enough so that making the second signal the primary target and forgoing the first target is more profitable.

If the second signal is hit exactly two out of three times, all three strategies produce the same return. If the second signal is hit less than two out of three times, all-in on the first signal still makes twelve dollars

-- more than any other strategy. If it is hit more than two out of every three times, betting it all on the second signal always makes the most money.

If we use strategy number one, we always make four dollars every time the indicator hits two. That means for three trades, we make twelve dollars.

If we use strategy number two, we could make nothing in the reasonably probable case the indicator never hit the second level, but we could also make eighteen if we hit the second level in three out of three chances. Strategy number two has a bigger win side, but a wide variation. The level two indicator has to be hit at least two out of three times for this to be a better bet than the all-in on the first signal.

For the averaging-in strategy to do better than twelve dollars in three trades, it has to hit the second signal more than two out of three trades as well.   That would make a profit of five dollars on the two second signal events, and two dollars on the first signal that never grew to a second level signal.

If the intermediate signal is one third as common as the opening signal, with averaging-in you make two dollars twice and five dollars once out of three trades -- nine dollars on three trades. And averaging-in makes six dollars (half as much the all-in on the first signal)

A General Demonstration that Averaging-in Fails

Suppose we have a trading system composed of an opening signal, a closing signal, and set of intermediate signals. Let *p* be the share price at the opening signal, *p* - *δ* the price at the intermediate signal, and *c* the price at the closing signal. We show here that in general for a trading system with an intermediate signal, averaging-in is subordinated to an all-in strategy at the opening signal when the proportion *R* of the trading sequences with intermediate signals is less than R_γ, where

$$R_\gamma = \frac{E\left(\frac{c}{p}\right)}{E\left(\frac{c}{p-\delta}\right)}$$

and is similarly subordinated to an all-in strategy at the intermediate signal when greater than R_γ. Averaging-in is precisely equal to both all-in strategies only when $R = R_\gamma$, but is nowhere better. We are forced to conclude that averaging-in is never better than an all-in strategy.

Let m_0 be the total account available to the trading system, m_1 the portion of the account selected for the opening signal, and $m_0 - m_1$ selected for the intermediate signal.   Let R be the proportion of the trading sequences containing an intermediate signal. The expected proceeds from averaging-in are then

$$P_a = E(proceeds|averaging\ in) = R\left[m_1 E\left(\frac{c}{p}\right) + (m_0 - m_1)E\left(\frac{c}{p-\delta}\right)\right] + (1-R)m_1 E\left(\frac{c}{p}\right)$$

$$= R(m_0 - m_1)E\left(\frac{c}{p-\delta}\right) + m_1 E\left(\frac{c}{p}\right)$$

If instead all the money is put on the opening signal, the expected proceeds are

$$P_o = E(proceeds|all\ on\ opening\ signal) = m_0 E\left(\frac{c}{p}\right)$$

And if all of the money is put on the intermediate signal

$$P_i = E(proceeds|all\ on\ intermediate\ signal) = Rm_0 E\left(\frac{c}{p-\delta}\right)$$

We now show that there is no averaging-in strategy that is better than putting all the money at play at once:

i. if $P_a > P_o$ then $R > R_\gamma$
ii. but if $R \geq R_\gamma$ then $P_i \geq P_a$ and in all cases, the proceeds from strategies for putting all the money into play at once, P_o or P_i are greater than or equal to that of the averaging-in strategy.

(i)

Let

$$P_a > P_o$$

$$R(m_0 - m_1)E\left(\frac{c}{p-\delta}\right) + m_1 E\left(\frac{c}{p}\right) > m_0 E\left(\frac{c}{p}\right)$$

Substituting

$$E\left(\frac{c}{p}\right) = R_\gamma E\left(\frac{c}{p-\delta}\right)$$

and

$$R(m_o - m_1) + m_1 R_\gamma > m_0 R_\gamma$$

$$R > R_\gamma$$

(ii)

$$P_a = m_0 RE\left(\frac{c}{p-\delta}\right) - m_1 RE\left(\frac{c}{p-\delta}\right) + m_1 E\left(\frac{c}{p}\right)$$

$$= P_i - m_1 RE\left(\frac{c}{p-\delta}\right) + m_1 E\left(\frac{c}{p}\right)$$

And solving for *R*

$$R = \frac{1}{m_1 E\left(\frac{c}{p-\delta}\right)}\left[P_i - P_a + m_1 E\left(\frac{c}{p}\right)\right]$$

Recalling that

$$R_\gamma = \frac{E\left(\frac{c}{p}\right)}{E\left(\frac{c}{p-\delta}\right)}$$

Then if

$$R \geq R_\gamma$$

$$\frac{1}{m_1 E\left(\frac{c}{p-\delta}\right)}\left[P_i - P_a + m_1 E\left(\frac{c}{p}\right)\right] \geq \frac{E\left(\frac{c}{p}\right)}{E\left(\frac{c}{p-\delta}\right)}$$

$$P_i - P_a + m_1 E\left(\frac{c}{p}\right) \geq m_1 E\left(\frac{c}{p}\right)$$

$$P_i \geq P_a$$

Q.E.D.

We conclude, for a trading system with intermediate signals, that the best strategy is to keep a rolling estimate of R_γ and a rolling estimate of *R*, the ratio of trading sequences containing an intermediate signal. When $R < R_\gamma$ put all of the account on the opening signal, and when $R > R_\gamma$ plan to put all of the money on the intermediate signal following the opening signal rather than on the opening signal itself.

Multiple Intermediate Signals

Similar arguments can be made for multiple intermediate signals. For two intermediate signals define

$$R_{\gamma_1} = \frac{E\left(\frac{c}{p}\right)}{E\left(\frac{c}{p-\delta_1}\right)}$$

$$R_{\gamma_2} = \frac{E\left(\frac{c}{p}\right)}{E\left(\frac{c}{p-\delta_2}\right)}$$

Again keep rolling estimates of R_{γ_1}, R_{γ_2}, and *R*, and when $R < R_\gamma$ plan to go all-in on the opening signal, and when $R_{\gamma_1} < R < R_{\gamma_2}$ plan to go all-in on the first intermediate signal, and when $R > R_{\gamma_2}$plan to go all-in on the second intermediate signal.

An Averaging-In System Example

In Chapter 8 of *High Probability ETF Trading*, Connors and Alvarez [1] present a trading system that includes averaging-in. The system as is follows

1. The ETF is above the 200 day moving average
2. If the RSI with period 2 is below 25 for two days in a row, use 10% of the account to purchase shares.
3. If the price is lower than the opening price, use 20% of the account to purchase shares.
4. If the price is again lower, use 30% of the account to purchase shares.
5. If the price is again lower, use the remaining 40% of the account to purchase shares.
6. Exit when the RSI with period 2 closes above 70.

This trading system was simulated for the SPY over the interval from Jan 29, 1993 through Oct 12, 2009. The simulation included an account and took into consideration the bid/asked spread and transaction costs. The results are

Profit	325599.11
Opening trades	202
Total Trades	410
Winning trades	182
Losing trades	20
Win percent	90.1
1st intermediate trades	106
2nd intermediate trades	69
3rd intermediate trades	33
4th intermediate trades	0
1st delta	.8054
2nd delta	1.203
3rd delta	1.7724
4th delta	0
$E(c/p)$	1.0044
$E(c/(c-\delta_1)$	.9978
$E(c/p)/E(c/(c-\delta_1)$	1.0067
$E(c/(c-\delta_2)$	.9953
$E(c/p)/E(c/(c-\delta_2)$	1.0092
$E(c/(c-\delta_3)$	.9894
$E(c/p)/E(c/(c-\delta_3)$	1.0152

All of the ratios are greater than one indicating that the strategy of all-in on the opening signal is the only successful strategy.

Re-running the simulation where all the money is put on the opening signal we get

Profit	653755.62
Trades	202
Winning trades	157
Losing trades	45
Win percent	77.72

As expected, the profit is greater even though the percent winning trades is smaller.

Toward An Optimal System

An alternative method for evaluating averaging-in is to apply the Design of Experiments method to the trading system and see what is found for the proportions for averaging-in. An eight factor design was generated for the following trading system

1. The ETF is above the 200 day moving average
2. If the RSI with period *factor 5* is below *factor 6* for two days in a row, use *factor 1*% of the account to purchase shares.
3. If the price is lower than the opening price, use *factor 2*% of the account to purchase shares.
4. If the price is again lower, use *factor 3*% of the account to purchase shares.
5. If the price is again lower, use the remaining *factor 4*% of the account to purchase shares.
6. Exit when the RSI with period *factor 7* closes above *factor 8*.

Factors 1 through 4 are mixture variables that are constrained to the unit interval and sum to 1. Factors 5 and 7 are integers with values on the interval [2,6], and factors 6 and 8 are process variables, i.e., continuous, with intervals [5,40] and [55,95] respectively. For details concerning the conduct of a Design of Experiments as applied to a technical trading system see [2] and [3].

The results of the analysis are the following settings,

Factor 1	1.000
Factor 2	0.000
Factor 3	0.000
Factor 4	0.000
Factor 5	2
Factor 6	40.0000
Factor 7	6
Factor 8	66.1380

The optimal settings chose to put all of the account into the opening signal. These settings produced the following outcome,

Profit	1772574.87
Trades	195
Winning trades	140
Losing trades	55
Win percent	71.79

Discussion

The Design of Experiments involved 164 trials at parameter settings distributed throughout the parameter space in satisfaction of a mathematical criterion unrelated to how well they might generate a profit. It is a testimony to the robustness of the trading system that only 10% of the trials produced a loss.

Nor should it be held against the trading system that its profit represents such a small return relative to the time duration of 13 years. As presented in [1] the system is applied to a larger set of ETF's with relatively uncorrelated pricing movements where the total returns from all ETF's would be more impressive.

What has been demonstrated here is not that the trading system fails to generate profits but that without averaging-in greater profits are produced.

References

[1] Connors, L. and C. Alvarez, *High Probability ETF Trading*, TradingMarkets Publishing Group, 2009.

[2] Schoenberg, Ronald J., "Application of Design of Experiments to an Automated Trading System", *Futures*, February, 2010.

[3] Schoenberg, Ronald J., "Follow-up Study of an Application of Design of Experiments to a Technical Trading System", http://www.optionbots.com/DOE/follow-up.pdf, 2009.